Christ Church Grammar School

MATHEMATICS SPECIALIST

2016 YEAR 12 PROGRAM

OVERVIEW

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

AIMS

The Mathematics Specialist ATAR course aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

YEAR 12 PROGRAM

This program is based on A. J. Sadler's Mathematics Methods Units 3 & 4, but references will also be made to other sources. The time frame is used as a guide only and may be adjusted to suit the progress of individual classes. The assessment dates are final and all students are to sit assessments on the allocated dates and times. Students who are absent for an assessment will be required to present the Studies Office with a Medical Certificate upon their return to school.

ASSESSMENT:

There will be 5 Tests, 3 Investigations, a Semester 1 Examination based on Unit 1 and a Semester 2 Examination based on Units 3 and 4. Marks and grades will be recorded as the mark and grade for Mathematics Specialist Year 12. Tests and examinations will include a Calculator Free section comprising one-third of the assessment and a Calculator Assumed section comprising two-thirds of the assessment.

ADDITIONAL RESOURCES:

Mathematics Specialist 12	Nelson
Mathematics Specialist Units 3 & 4 Revision Series	O.T. Lee
Sample Examination Papers	Various authors and schools

HOMEWORK and EXPECTATION:

All exercises in A. J. Sadler's Mathematics Specialist Units 3 & 4 are to be completed, with relevant problems to be attempted promptly after each lesson. All questions in O. T. Lee's Mathematics Specialist Units 3 & 4 Revision Series are to be attempted at home as part of the student's revision and preparation for tests. Sample examination papers with full solutions will be provided to help students prepare for examinations.

STRUCTURE & CONTENT

The Year 12 syllabus is divided into two units, each of approximately one semester duration.

Unit	Topic	Sub-topics	Duration
3	3.1: Complex numbers	• Cartesian forms • Complex arithmetic using polar form • The complex plane (The Argand plane) • Roots of complex numbers • Factorisation of polynomials	18 hours
	3.2: Functions and sketching graphs	• Functions • Sketching graphs	16 hours
	3.3: Vectors in three dimensions	• The algebra of vectors in three dimensions • Vector and Cartesian equations • Systems of linear equations • Vector calculus	21 hours
4	4.1: Integration and applications of integration	• Integration techniques • Applications of integral calculus	20 hours
	4.2: Rates of change and differential equations	• Applications of differentiation • Modelling motion	20 hours
	4.3: Statistical inference	• Sample means • Confidence intervals for means	15 hours

COURSE PROGRAM

Week	Syllabus reference, content	Text	Assessment
Term 1			
Week 1	**Cartesian forms** 3.1.1 review real and imaginary parts Re(z) and Im(z) of a complex number z 3.1.2 review Cartesian form 3.1.3 review complex arithmetic using Cartesian forms	**Sadler** Unit 3 Ch 1A	
1 – 2	**Complex arithmetic using polar form** 3.1.4 use the modulus |z| of a complex number *z* and the argument Arg(z) of a non-zero complex number z and prove basic identities involving modulus and argument 3.1.5 convert between Cartesian and polar form 3.1.6 define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these 3.1.7 prove and use De Moivre's theorem for integral powers	**Sadler** Unit 3 Ch 2A-C, 2F	
3	**The complex plane (The Argand plane)** 3.1.8 examine and use addition of complex numbers as vector addition in the complex plane 3.1.9 examine and use multiplication as a linear transformation in the complex plane 3.1.10 identify subsets of the complex plane determined by relations such as $\lvert z-3i\rvert \leq 4$, $\frac{\pi}{4} \leq Arg(z) \leq \frac{3\pi}{4}$ and $\lvert z-1\rvert = 2\lvert z-i\rvert$	**Sadler** Unit 3 Ch 2D	
4	**Roots of complex numbers** 3.1.11 determine and examine the nth roots of unity and their location on the unit circle 3.1.12 determine and examine the nth roots of complex numbers and their location in the complex plane	**Sadler** Unit 3 Ch 2E	**Test 1** Unit 3 Ch 1A, 2A-F
5	**Factorisation of polynomials** 3.1.13 prove and apply the factor theorem and the remainder theorem for polynomials 3.1.14 consider conjugate roots for polynomials with real coefficients 3.1.15 solve simple polynomial equations	**Sadler** Unit 3 Ch 1B	
6 – 7	**Functions** 3.2.1 determine when the composition of two functions is defined 3.2.2 determine the composition of two functions 3.2.3 determine if a function is one-to-one 3.2.4 find the inverse function of a one-to-one function 3.2.5 examine the reflection property of the graphs of a function and its inverse	**Sadler** Unit 3 Ch 3A-B	**Test 2** Unit 3 Ch 1B, 3A-B

Week	Syllabus reference, content	Text	Assessment
Term 1 Weeks 8 – 9	**Sketching graphs** 3.2.6 use and apply $\lvert x \rvert$ for the absolute value of the real number x and the graph of $y = \lvert x \rvert$ 3.2.7 examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$ 3.2.8 sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree	**Sadler** Unit 3 Ch 3C-D	**Investigation 1**
10	**The algebra of vectors in three dimensions** 3.3.1 review the concepts of vectors from Unit 1 and extend to three dimensions, including introducing the unit vectors **i**, **j** and **k** 3.3.2 prove geometric results in the plane and construct simple proofs in 3 dimensions	**Sadler** Unit 3 Ch 4A-F	
Term 2			
Weeks 1 – 2	**Vector and Cartesian equations** 3.3.3 introduce Cartesian coordinates for three dimensional space, including plotting points and equations of spheres 3.3.4 use vector equations of curves in two or three dimensions involving a parameter and determine a 'corresponding' Cartesian equation in the two-dimensional case 3.3.5 determine a vector equation of a straight line and straight line segment, given the position of two points or equivalent information, in both two and three dimensions 3.3.6 examine the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet 3.3.7 use the cross product to determine a vector normal to a given plane 3.3.8 determine vector and Cartesian equations of a plane	**Sadler** Unit 3 Ch 5A-E	
2 – 3	**Systems of linear equations** 3.3.9 recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations 3.3.10 examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables	**Sadler** Unit 3 Ch 6A-B	**Test 3** Unit 3 Ch 5A-E, 6A-B
4 – 5	**Vector calculus** 3.3.11 consider position vectors as a function of time 3.3.12 derive the Cartesian equation of a path given as a vector equation in two dimensions, including ellipses and hyperbolas 3.3.13 differentiate and integrate a vector function with respect to time 3.3.14 determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration 3.3.15 apply vector calculus to motion in a plane, including projectile and circular motion	**Sadler** Unit 3 Ch 7A-B	**Investigation 2**

Week	Syllabus reference, content	Text	Assessment
Term 2 Weeks 6 – 7	Exam Period		**Semester 1 Examination**
8 – 9	**Integration techniques** 4.1.1 integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}\left(1-\cos 2x\right)$, $\cos^2 x = \frac{1}{2}\left(1+\cos 2x\right)$ and $1+\tan^2 x = \sec^2 x$ 4.1.2 use substitution u = g(x) to integrate expressions of the form $f(g(x))\,g'(x)$ 4.1.3 establish and use the formula $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$ for $x \neq 0$ 4.1.4 use partial fractions where necessary for integration in simple cases	**Sadler** Unit 4 Ch 9A-E	
9 – 10	**Applications of integral calculus** 4.1.5 calculate areas between curves determined by functions 4.1.6 determine volumes of solids of revolution about either axis 4.1.7 use technology with numerical integration	**Sadler** Unit 4 Ch 9F	**Test 4** Unit 3, Ch 7A-B Unit 4, Ch 9A-F

Week	Syllabus reference, content	Text	Assessment
Term 3			
Weeks 1 – 3	**Applications of differentiation** 4.2.1 use implicit differentiation to determine the gradient of curves whose equations are given in implicit form 4.2.2 examine related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ 4.2.3 apply the incremental formula $\partial y \approx \frac{dy}{dx} \partial x$ to differential equations 4.2.4 solve simple first order differential equations of the form $\frac{dy}{dx} = f(x)$; differential equations of the form $\frac{dy}{dx} = g(y)$; and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$, using separation of variables 4.2.5 examine slope (direction or gradient) fields of a first order differential equation 4.2.6 formulate differential equations, including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved	**Sadler** Unit 4 Ch 8A-D **Sadler** Unit 4 Ch 10A-C	**Investigation 3**
3 – 4	**Modelling motion** 4.2.7 consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ for acceleration	**Sadler** Unit 4 Ch 11A-B	
5	**Sample means** 4.3.1 examine the concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ 4.3.2 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\bar{X}$ across samples of a fixed size *n*, including its mean μ its standard deviation $\frac{\sigma}{\sqrt{n}}$ (where μ and σ are the mean and standard deviation of *X*), and its approximate normality if *n* is large 4.3.3 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where *s* is the sample standard deviation	**Sadler** Unit 4 Ch 12A	

Week	Syllabus reference, content	Text	Assessment
Term 3 Weeks 6 – 7	**Confidence intervals for means** 4.3.4 examine the concept of an interval estimate for a parameter associated with a random variable 4.3.5 examine the approximate confidence interval $\left(\bar{X}-\frac{zs}{\sqrt{n}},\bar{X}+\frac{zs}{\sqrt{n}}\right)$ as an interval estimate for the population mean μ, where z is the appropriate quantile for the standard normal distribution 4.3.6 use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ 4.3.7 use $\bar{x}$ and s to estimate μ and σ to obtain approximate intervals covering desired proportions of values of a normal random variable, and compare with an approximate confidence interval for μ	**Sadler** Unit 4 Ch 12B-C	**Test 5** Unit 4, Ch 8A-D, 10A-C, 11A-B, 12A
8 – 9	Revision		
10	Exam Period		**Semester 2 Examination**
Term 4			
Weeks 1 – 2	Revision, a detailed revision program will be provided.		

ASSESSMENT SCHEDULE

Assessment	Weighting	Topic guide	Date
Term 1			
Test 1	8%	Unit 3, chapters 1A, 2A-F	Week 4 Friday 26 February
Test 2	8%	Unit 3, chapters 1B, 3A-B	Week 7 Friday 18 March
Investigation 1	6%		Week 10 Friday 8 April
Term 2			
Test 3	8%	Unit 3, chapter 5A-E, 6A-B	Week 3 Friday 13 May
Investigation 2	7%		Week 4 Friday 20 May
Semester 1 Exam	15%	All topics from Unit 3	Weeks 6-7
Test 4	8%	Unit 3, chapters 7A-B Unit 4, chapters 9A-F	Week 10 Friday 1 July
Term 3			
Investigation 3	7%		Week 3 Friday 5 August
Test 5	8%	Unit 4, chapters 8A-D, 10A-C, 11A-B, 12A	Week 7 Wednesday 31 August
Semester 2 Exam	25%	All topics from Units 3 & 4	

Note: The assessment schedule is only to be used as a guide and is subject to change by the course coordinator. Students and subject teachers will be emailed closer to the date regarding the content of each assessment item with greater detail.

ASSESSMENT TABLE

Type of assessment	Weighting
Response Students respond using knowledge of mathematical facts, concepts and terminology, applying problem solving skills and algorithms. Tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions.	40%
Investigation Students plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of general inquiry skills, course-related knowledge and skills, and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to open-ended questions that require responses at the highest level of conceptual thinking. Students may be asked questions of an investigative nature for which they may need to communicate findings, generalise, or make and test conjectures. Typically conducted at the end of each semester and/or unit and reflecting the examination design brief for this syllabus.	40%

EXAMINATION DESIGN BRIEF

This examination consists of two sections.

Section One: calculator-free

Time allowed
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Permissible items
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: nil

Additional information
Changeover period during which the candidate is not permitted to work: up to 15 minutes

Section Two: calculator-assumed

Time allowed
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Permissible items
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Provided by the supervisor
A formula sheet

Additional information
It is assumed that candidates sitting this examination have a calculator with CAS capabilities for Section Two.
The examination assesses the syllabus content areas using the following percentage ranges. These apply to the whole examination rather than individual sections.

Content area	Percentage of exam
Complex numbers	15–20%
Functions and graphs	10–15%
Vectors in three dimensions	15–20%
Integration and application of integration	15–20%
Rates of change and differential equations	15–20%
Statistical inference	10–15%

The candidate is required to demonstrate knowledge of mathematical facts, conceptual understandings, use of algorithms, use and knowledge of notation and terminology, and problem-solving skills.
Questions can require the candidate to investigate mathematical patterns, make and test conjectures, and generalise and prove mathematical relationships. Questions can require the candidate to apply concepts and relationships to unfamiliar problem-solving situations, choose and use mathematical models with adaptations, compare solutions, and present conclusions. A variety of question types that require both open and closed responses can be included.
Instructions to candidates indicate that, for any question or part question worth more than two marks, valid working or justification is required to receive full marks.

Section	Supporting information
Section One: calculator-free 35% of the total examination 5–10 questions Suggested working time: 50 minutes	Questions examine content and procedures that can reasonably be expected to be completed without the use of a calculator i.e. without undue emphasis on algebraic manipulations or time consuming calculations. The candidate is required to provide answers that include: calculations, tables, graphs, interpretation of data, descriptions and/or conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media.
Section Two: calculator-assumed 65% of the total examination 8–13 questions Suggested working time: 100 minutes	Questions examine content and procedures for which the use of a calculator is assumed. The candidate can be required to provide answers that include: calculations, tables, graphs, interpretation of data, descriptions and/or conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media. The candidate can be required to investigate theoretical situations involving mathematical concepts and relationships, for which they need to generalise, construct proofs and make conjectures. The candidate can be required to solve problems from unfamiliar situations, choosing and using mathematical models with adaptations where necessary, comparing their solutions with the situations concerned, and then presenting their findings in context.

GRADE DESCRIPTIONS

A	**Identifies and organises relevant information** Identifies and organises relevant information from complex and scattered sources, such as the key parameters of a simple harmonic motion function, and identifies the correct solution of a trigonometric equation from the multiple possible solutions. Describes linear motion in three-dimensional spaces. Uses answers from previous parts of a problem to carry through and solve subsequent problems.
	Chooses effective models and methods and carries through the methods correctly Solves unstructured problems by choosing the most appropriate algebraic, vector or calculus techniques. Chooses efficient methods when dealing with derivatives, integrals and statistical inference. Clearly sets out the deductive reasoning and accurately carries it through. Simplifies complicated fractions and works efficiently with algebraic expressions in fraction form. Uses well-constructed diagrams and makes appropriate geometric connections when carrying out vector proofs.
	Obeys mathematical conventions and attends to accuracy Uses the correct notation at all times with vectors, matrices, functions and calculus. Sets out reasoning in clearly defined steps that are easily followed. Draws diagrams and graphs, including the polar form, with appropriate scales and labels. Works easily with exact values such as surds, radian values or natural exponentials e^x and recognises the difference between open and closed intervals.
	Links mathematical results to data and contexts to reach reasonable conclusions Pays attention to the units, gives exact value answers, uses the correct degree of accuracy and uses radian measure when appropriate. Always takes account of the domain as defined in the problem, or by the context of the question, and excludes any results outside it. Links polar solutions of the complex equation $z^n=c$ and the principal domain $-\pi < \theta \leq \pi$.
	Communicates mathematical reasoning, results and conclusions Sets out each step of deductive reasoning in a clear and logical sequence. Defines variables associated with diagrams and uses them consistently in the working of a problem. Carries a deductive proof through by working with only one side of the equation. Communicates the main steps of integration problems using the substitution method, including trigonometric identities. Relates the result of a problem to the context of the question by using the correct units and any related notation such as vector notation.

B	**Identifies and organises relevant information** Identifies and organises relevant information from dense sources, for example, descriptive passages, labelled diagrams, or recognises an advanced integration method in the form $\int (f'(x)/f(x))\,dx$. Recognises that for various equations, multiple angle solutions are possible in a given domain, for example, $\sin 2\theta = \frac{1}{2}$ for $0 \leq \theta \leq 2\pi$. Identifies key information from scattered sources, for example, using answers from previous parts of a problem and bringing them together to solve subsequent problems.
	Chooses effective models and methods and carries the methods through correctly Chooses and uses the correct technique or model in unpractised situations with vectors, functions, simple harmonic motion and the calculus of trigonometric functions. Uses well-constructed diagrams and makes appropriate geometric connections when carrying out vector proofs. Translates between representations in unpractised ways, connecting diagrams to vector algebra using correct notation.
	Obeys mathematical conventions and attends to accuracy Uses the correct notation with vectors, matrices, functions and calculus on most occasions. Obeys conventions, such as using appropriate limits of integration when integrating by substitution, and setting out the reasoning in clearly defined steps. Draws clear diagrams and graphs with appropriate scales and labels. Uses exact values, such as surds and radian values using π, and recognises the difference between open and closed intervals.
	Links mathematical results to data and contexts to reach reasonable conclusions Pays attention to the units, gives answers to the correct degree of accuracy and uses radian measure on most occasions. Takes account of the domain as defined in the problem and excludes results outside it.
	Communicates mathematical reasoning, results and conclusions Carries through calculations and simplifications in a clear and logical sequence. Defines variables associated with diagrams and uses them consistently in the working of a problem. Communicates the main steps of integration problems using the substitution method. Relates the result of a problem to the context of the question by using the correct units and any related notation.

C	**Identifies and organises relevant information from information that is relatively narrow in scope** Identifies and organises information from relatively narrow sources, for example, identifies the amplitude and period of a trigonometric function, plots simple graphs in the complex number plane and interprets given vector diagrams. Identifies appropriate rules in calculus, such as the product, quotient and chain rules for derivatives of exponential, logarithmic and trigonometric functions.
	Chooses effective models and methods and carries through the methods correctly Applies mathematical algorithms in practised ways, for example, the product, quotient and chain rules for derivatives of exponential, logarithmic and trigonometric functions. Integrates functions with less complex algebraic expressions. Uses a familiar trigonometric identity to simplify an expression, or uses implicit differentiation that is narrow in scope. Uses given vector diagrams effectively to answer simple questions.
	Obeys mathematical conventions and attends to accuracy Obeys mathematical conventions when simplifying an algebraic expression or working with an equation. Defines the correct interval for inequalities and translates simple inequalities to graphical intervals. Obeys mathematical conventions for sketching graphs and clearly labels the main features, for example, asymptotes. Applies the conventions for vector diagrams and complex number graphs. Translates between complex number representations in practised ways.
	Links mathematical results to data and contexts to reach reasonable conclusions Gives exact values and attends to units in short responses on most occasions. Includes implied units with the solution, such as metres per second or radians per second.
	Communicates mathematical reasoning, results and conclusions Shows adequate working and justifies answers with simple or routine statements. Relates the working to any vector diagram that has been given as part of the question. Makes clear sketches of simple functions, including required features such as intercepts of graphs. Draws simple diagrams to help solve problems with vectors and applications of calculus.

D	**Identifies and organises relevant information** Identifies and organises relevant information that is narrow in scope. Identifies the amplitude of a trigonometric function. Uses the derivative to get the gradient at a point.
	Chooses effective models and methods and carries the methods through correctly Answers familiar structured questions that require short responses, for example, calculating the constant c in $y = mx + c$ given the gradient m and a point . Completes implicit differentiation with straightforward expressions. Integrates functions with simple algebraic expressions. Makes common sense connections. Recognises simple complex number inequalities and draws the regions in the complex plane. Uses a calculator appropriately for straightforward calculations, algebra and graphing. Multiplies matrices accurately.
	Obeys mathematical conventions and attends to accuracy Applies conventions for function notation. Obeys mathematical conventions by including a constant of integration. Defines linear equations and applies conventions for diagrams and graphs. Draws the complex numbers in correct position in the plane. Labels axes, shows scale and labels graphs and intersection points.
	Links mathematical results to data and contexts to reach reasonable conclusions On occasions, recognises specified conditions and attends to units in short responses. Sometimes includes units in the solution.
	Communicates mathematical reasoning, results and conclusions Shows some working. Sets calculations out in a manner that can be checked for accuracy.

E	Does not meet the requirements of a D grade.